International
Grammar School

Year 12
Mathematics
Trial HSC Examination
2016

General Instructions

- Reading time – 5 minutes
- Working time – 3 hour
- Write using black or blue pen
- Board-approved calculators may be used
- A Reference Sheet is provided with this question paper
- In Questions 11 - 16, show relevant mathematical reasoning and/or calculations

Note: Any time you have remaining should be spent revising your answers.

Total marks – 100

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II

90 marks

- Attempt Questions 11 – 16
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover
- Allow about 2 hours 45 minutes for this section.

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Which of the following is the value of $3e^3$, correct to 3 significant figures?

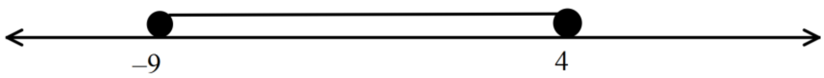
(A) 60.2

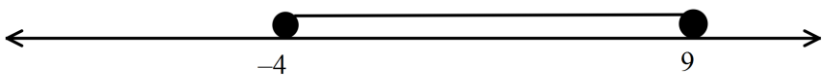
(B) 60.3

(C) 60.256

(D) 60.257

2 Which graph shows the solution to $|2x-5| \le 13$?

(A)


(B)


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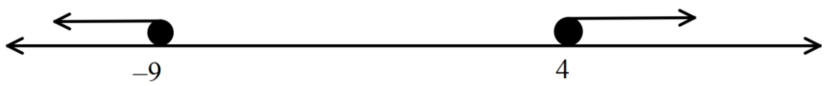
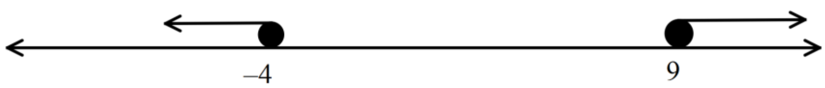


(D)


3 Which statement correctly describes the roots of $2x^2 + 4x - 5 = 0$

(A) The roots are equal, real and irrational.

(B) The roots are equal, real and rational.

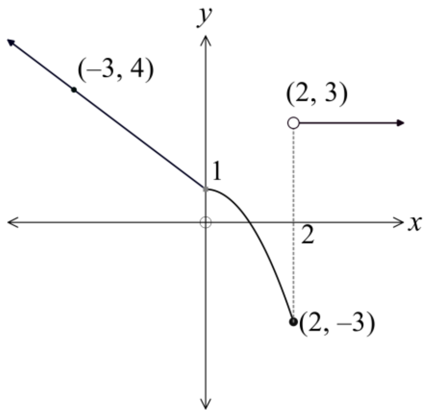
(C) The roots are unequal, real and irrational.

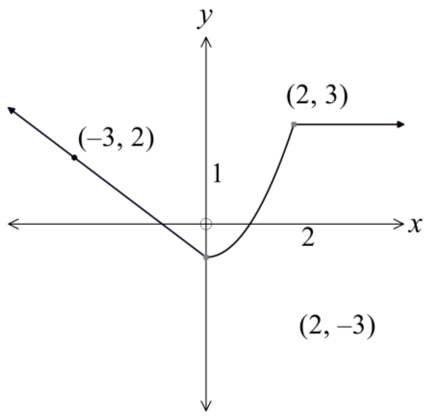
(D) The roots are unequal and unreal.

4 Which of the graphs would represent the function below?

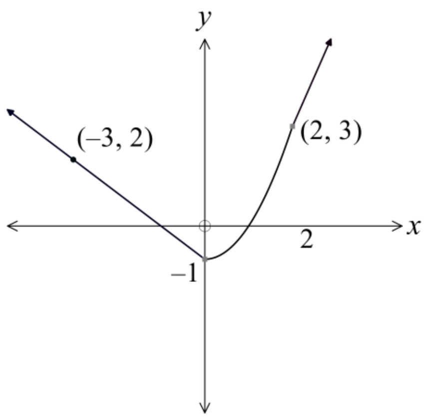
$$\begin{cases} y = 1 - x & x < 0 \\ y = 1 - x^2 & 0 \leq x \leq 2 \\ y = 3 & x > 2 \end{cases}$$

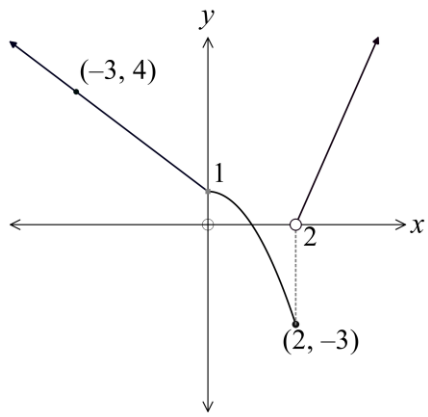
(A) (B)





(C) (D)





5 Given that $f(x) = \dfrac{4x^5 - 8x}{x^3}$, what is the value of $f'(2)$?

(A) 2

(B) 8

(C) 12

(D) 18

6 Which of the following is the same as $\operatorname{cosec}(\pi + \theta)$?

(A) $\dfrac{-1}{\sin\theta}$

(B) $\dfrac{-1}{\cos\theta}$

(C) $\dfrac{1}{\cos\theta}$

(D) $\dfrac{1}{\sin\theta}$

7 A bag contains 12 marbles. Four of the marbles are blue, two are white and the remainder are red. **Three marbles** are drawn from the bag.

What is the probability that all three marbles are red?

(A) $\dfrac{1}{55}$

(B) $\dfrac{1}{22}$

(C) $\dfrac{1}{11}$

(D) $\dfrac{5}{22}$

8 Use Simpsons Rule with three function values to approximate: $\displaystyle\int_e^{3e} ln\, x\, dx$.

(A) $\dfrac{2}{3e}$

(B) $\dfrac{e(4ln(5) + 3)}{6}$

(C) $\dfrac{e(4ln(6) + 6)}{3}$

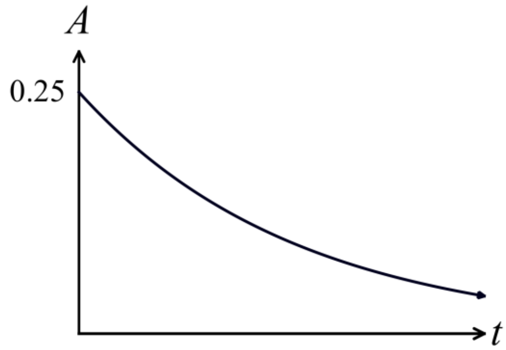
(D) $\dfrac{e(ln(48) + 6)}{3}$

9 The amount of a substance (A) is initially 20 units.

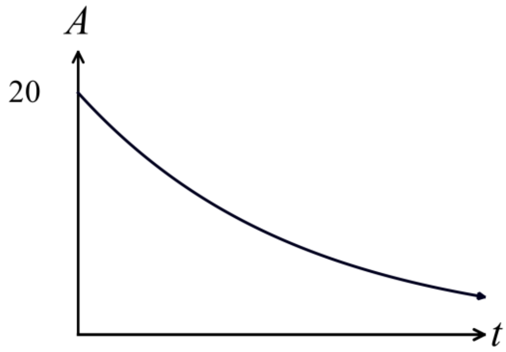
The rate of change in the amount is given by $\dfrac{dA}{dt} = 0.25A$.

Which graph shows the amount of the substance over time?

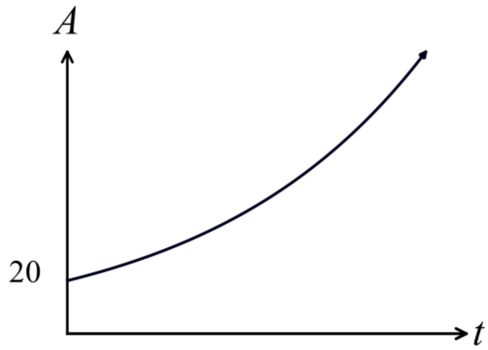
(A)



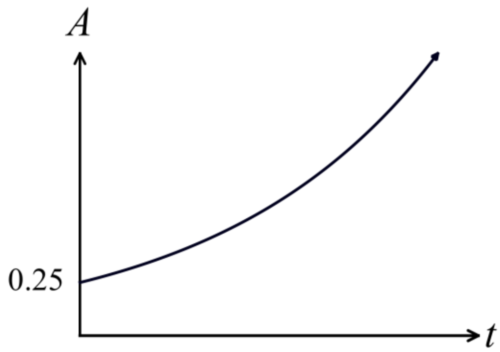
(B)



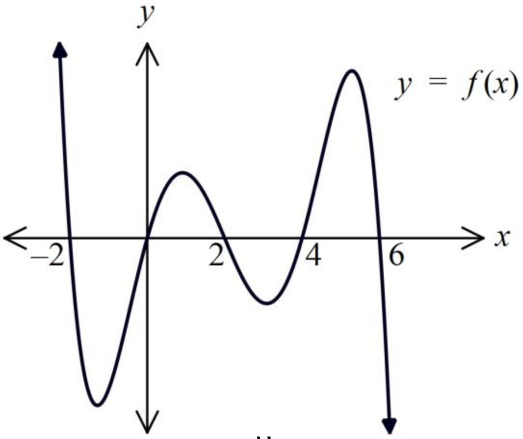
(C)



(D)

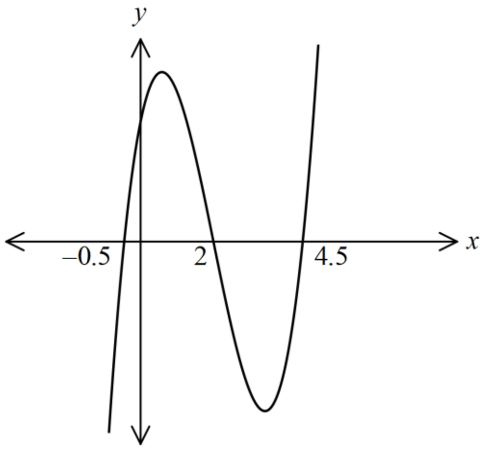


10 The graph of $y = f(x)$ is shown below.

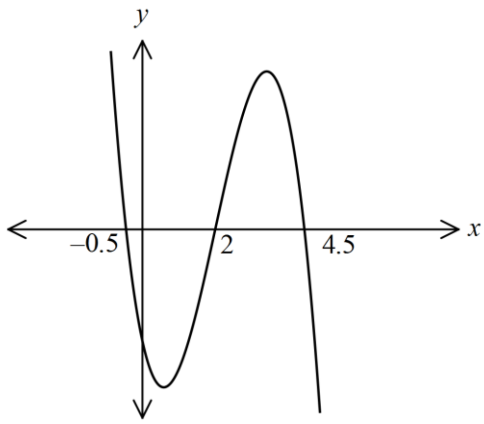


Which of these graphs could represent $y = f'(x)$?

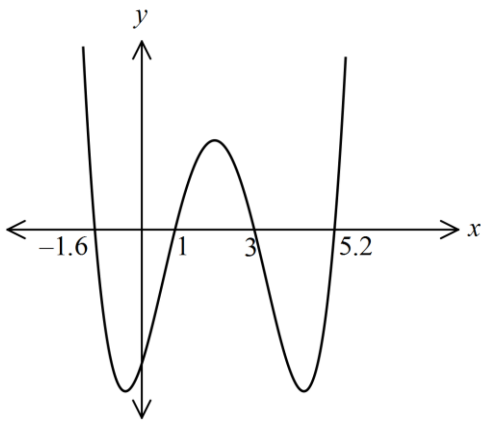
(A)



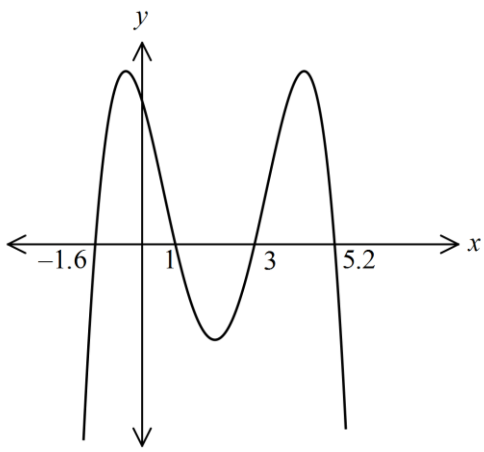
(B)



(C)



(D)



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Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Expand and simplify $\left(2\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)$. 2

(b) Simplify $\dfrac{a^3-b^3}{a^2-b^2}$ 2

(c) Find the equation of the tangent to the curve $y=(x^2-2)^4$ at the point where $x=1$. 2

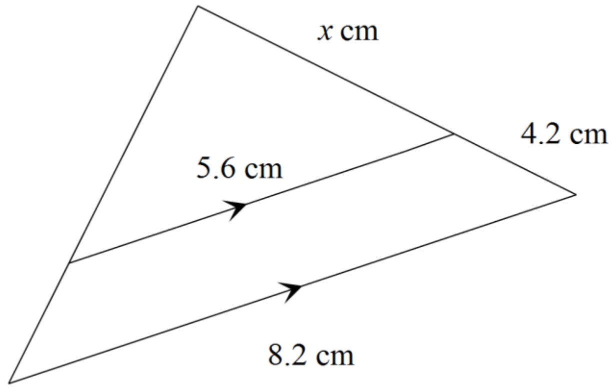
(d) Find $\displaystyle\int_2^4 \frac{6x^4-3x^3-1}{x^2}\,dx$. 3

(e) Differentiate $\sqrt{x}\,e^x$. 2

Question 11 continues on page 10

Marks

(f) Find the value of x (correct to the nearest mm). 2



(g) If α and β are the roots of $4x^2 - 5x - 1 = 0$ find the value of:

(i) $\alpha + \beta$ and $\alpha\beta$ 1

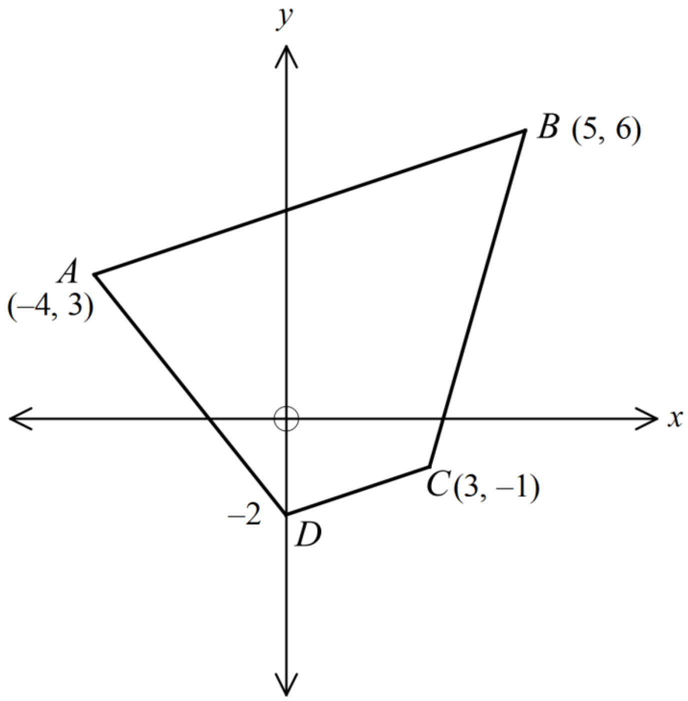
(ii) $\alpha^2\beta + \alpha\beta^2$ 1

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the value(s) of m for which the equation $x^2 + mx + (m+3) = 0$ has **real roots**. 2

(b) A quadrilateral is formed by the points $A(-4,3)$, $B(5,6)$ $C(3,-1)$ and $D(0,-2)$ as shown in the diagram



(i) Show that the quadrilateral is a trapezium, with $AB \parallel DC$. 2

(ii) Show that the equation of AB is $x - 3y + 13 = 0$. 1

(iii) Find the perpendicular distance from D to AB. 2

(iv) Find the area of the trapezium $ABCD$. 2

Question 12 continues on page 12

(c) The sales team at Frontier phone company sell 12 000 phones in the first month of operation. They increase their sales by 800 phones each month on the preceding month's sales.

(i) Find the number of phones sold in the last month of the second year of operation. 2

(ii) Find the number of phones sold over the entire two year period. 2

(iii) Sampson, another phone company commenced business at exactly the same time as Frontier. Their sales team sell 5 000 phones in their first month of operation and increase their sales by 1 500 each month on the preceding month's sales.

After how many months will both companies total sales become equal? 2

End of Question 12

Question 13 (15marks) Use a SEPARATE writing booklet **Marks**

(a) Given $f(x) = \log_{10} 2x$, find $f'(2)$ as an exact value. 3

(b)

(i) Show that $\frac{d}{dx}\left[\cos^3 3x\right] = -9\sin 3x \cos^2 3x$ 2

(ii) Hence, or otherwise, find $\int \sin 3x - \sin^3 3x \, dx$ 2

(c) The size of a colony of bees is given by the equation $P = 5000e^{kt}$ where P is the population after t weeks.

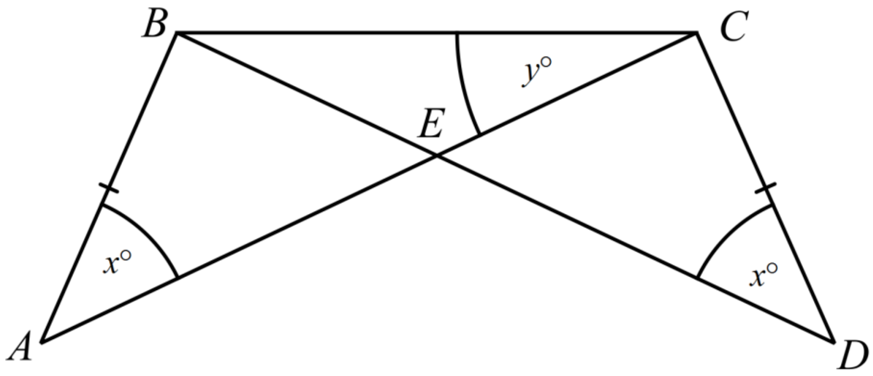
(i) If there are 6000 bees after one week, find the value of k to 2 decimal places. 1

(ii) When will the colony (to the nearest day) triple in size? 1

(iii) What is the growth rate of the population after two weeks? 2

Question 13 continues on page 14

(d) In the diagram below, $AB = CD$ and $\angle BAC = \angle CDB = x°$
Also $\angle BCA = y°$.



Copy or trace the diagram into your writing booklet.

(i) Prove that $\Delta ABE \equiv \Delta DCE$ 2

(ii) Show that $\angle ABE = 180° - (x + 2y)°$. 2

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) A particular curve passes through the point (2, 7).

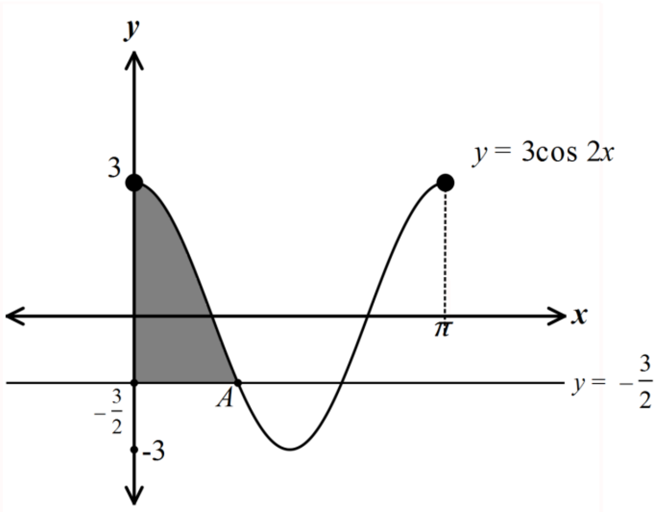
For this curve $\frac{dy}{dx} = 6e^{3x-6}$. Write down the equation of the curve. 2

(b)

(i) Find the exact values of u for which $2u^2 + \sqrt{3}u - 3 = 0$. 2

(ii) Hence or otherwise solve $2\cos^2 x + \sqrt{3}\cos x - 3 = 0$ for $0 \le x \le 2\pi$. 2

(c) The graphs of $y = 3\cos 2x$ and $y = -\frac{3}{2}$ are shown in the diagram below, where point A is the point of intersection of the two graphs and $0 \le x \le \pi$.



(i) Show that the x coordinate of point A is $\frac{\pi}{3}$ 1

(ii) Hence find the exact value of the shaded area that is enclosed by $y = 3\cos 2x$, $y = -\frac{3}{2}$ and the y-axis. 3

Question 14 continues on page 16

(d) Lola borrows \$300 000 to buy a house. The loan agreement is for interest rate of 6% p.a. compounded monthly. She agreed to repay the loan in 25 years with equal monthly repayments of $\$M$.

(i) Show that the amount owing after the second repayment is

$$A_2 = 300000(1.005)^2 - M(1+1.005)$$ 1

(ii) Calculate the monthly repayment M if the loan is paid off in 25 years. Give your answer to the nearest dollar. 2

(iii) If Lola doubles the amount of her monthly repayments, how much more quickly (in months) will she pay off the loan? 2

End of Question 14

Question 15 (15marks) Use a SEPARATE writing booklet **Marks**

(a) The point $P(x, y)$, moves so that it is equidistant from the points $A(-2,5)$ and $B(4,-7)$.

(i) Write an expression for AP^2. 1

(ii) Write the fully simplified equation that describes the locus of P. 2

(b) The acceleration of a particle moving along the x-axis is given by

$$\ddot{x} = 6t - 14$$

where x is the displacement from the origin in metres, t is the time in seconds and $t \geq 0$.

The particle is initially 2 m **to the left of the origin**, moving at 8 m/s toward the right.

(i) Find expressions for the velocity and displacement of the particle. 2

(ii) At what times is the particle at rest? 2

Question 15 continues on page 18

(c) Consider the function $f(x) = 1 - 3x + x^3$, in the domain $-2 \leq x \leq 3$.

(i) Find the coordinates of the turning points and determine their nature. 2

(ii) Find the coordinates of the point of inflexion. 1

(iii) Draw a neat half page sketch of the curve $y = f(x)$ clearly showing all its essential features, in the domain $-2 \le x \le 3$ 2

(iv) What is the maximum value of the function $f(x)$ in the domain $-2 \le x \le 3$? 1

(d) Find the exact value of the gradient of the tangent to the curve $y = x\ln x$ at the point where $x = e^x$. 2

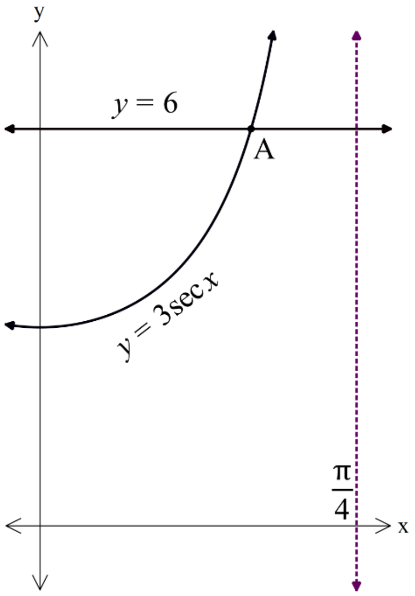
End of Question 15

Question 16 (15 marks) se a SEPARATE writing booklet **Marks**

(a) The function $y = ax^2 + bx + 4$ and its gradient function intersect at points where $x = 2$ and $x = 4$.

Find the value of a and b. 3

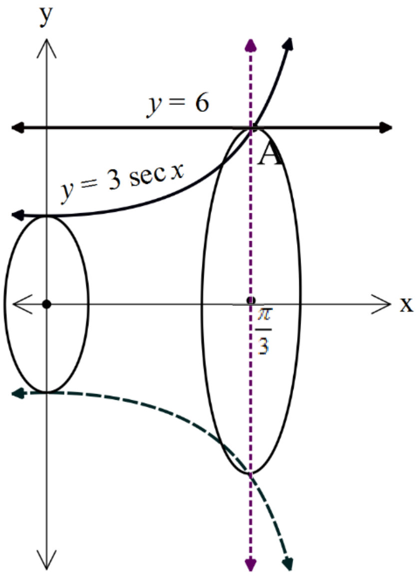
(b) The graph below shows the line $y = 6$ and the curve $y = 3\sec x$ for $0 \le x \le \dfrac{\pi}{4}$



(i) By solving the equation $3\sec x = 6$, show that the point A where the line and curve intersect has coordinates $\left(\dfrac{\pi}{3}, 6\right)$. 2

Question 16 continues on page 19

(ii) The region enclosed between the curve $y = 3\sec x$ and the x axis between $x = 0$ and $\frac{\pi}{3}$ is rotated about the x axis. 3



Find the exact volume of the solid formed.

(c)

(i) Find the coordinates of the focus, S , of the parabola $y = x^2 + 1$. 2

(ii) The graphs of $y = x^2 + 1$ and the line $y = x + k$ have only one point of intersection, P . Show that the x - coordinate of P satisfies 1

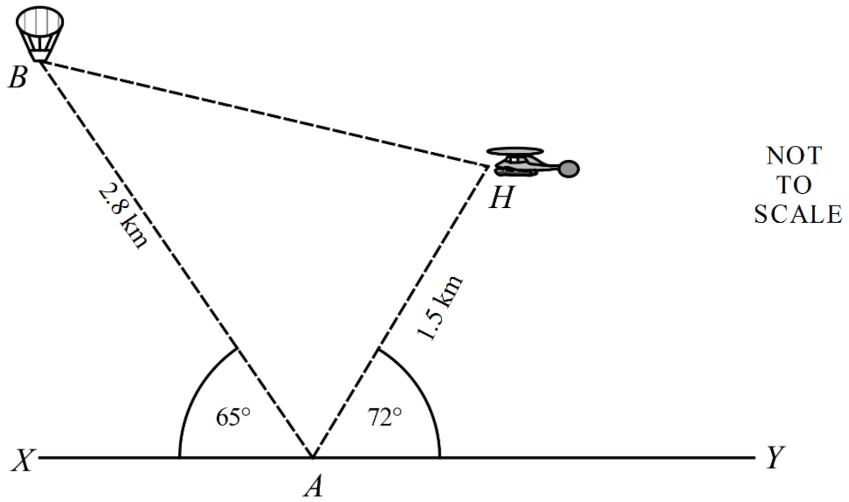
$$x^2 - x + 1 - k = 0$$

(iii) Using the discriminant, or otherwise, find the value of k . 1

Question 16 continues on page 20

(d) From a point on a level ground an observer sees a balloon B and a helicopter H which are both stationary at the time.

The balloon is positioned due west of point A, at a distance of 2.8 km on an angle of elevation of $65°$ and the helicopter is positioned due east of point A, at a distance of 1.5 km on an angle of elevation of $72°$, as shown in the diagram.



(i) Show that the distance between the helicopter and the balloon is approximately 2.0 km. 1

(ii) Find the bearing of the helicopter from the balloon. 2
Answer correct to the nearest degree.

End of Examination

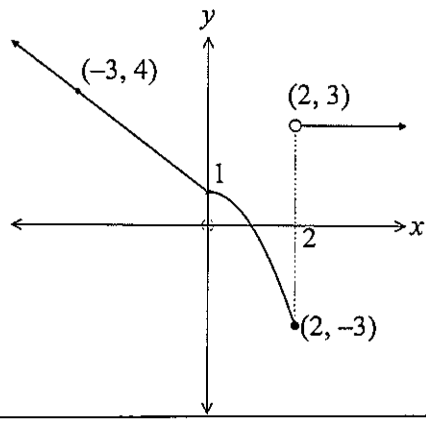
2016 Year 12 HSC Trial Examination **Student Name: . . . .**

Mathematics

Section A Multiple-Choice Answer Sheet

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

Multiple Choice Worked Solutions

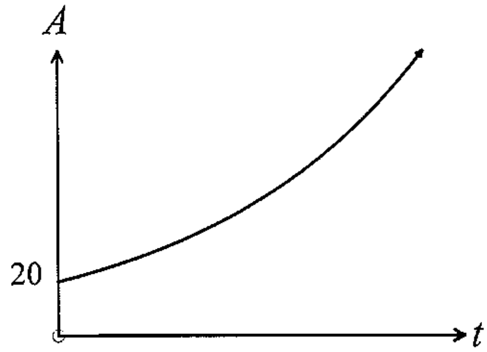
No	Working	Answer
1.	$3 \times e^3 = 60.2566\ldots$ $= 60.3$ (3 sf)	B
2	$\lvert 2x-5\rvert \le 13$ $-13 \le 2x-5 \le 13$ $-8 \le 2x \le 18$ $-4 \le x \le 9$  	**B**
3	$2x^2 + 4x - 5 = 0$ $\Delta = b^2 - 4ac$ $= 4^2 - 4(2)(-5)$ $= 16 + 40$ $= 56$ (which is positive and not a perfect square.) ∴ roots are unequal, real and irrational.	**C**
4	Graph for $x < 0$ is a straight line with a negative gradient and intercept of 1 on y axis, It does not include upper domain endpoint but it is common with next section. Graph for $0 \le x \le 2$ is a is a parabola which is concave down and has an intercept of 1 on y axis, includes both endpoints. Graph for $x > 2$ is a horizontal straight line through 3 on y axis,. does not include lower domain endpoint.  	**A**
5	$f(x) = \dfrac{4x^5 - 8x}{x^3}$ $= 4x^2 - 8x^{-2}$ $f'(x) = 8x + 16x^{-3}$ $f'(2) = 8(2) + 16(2)^{-3}$ $= 16 + \dfrac{16}{8}$ $= 18$	**D**

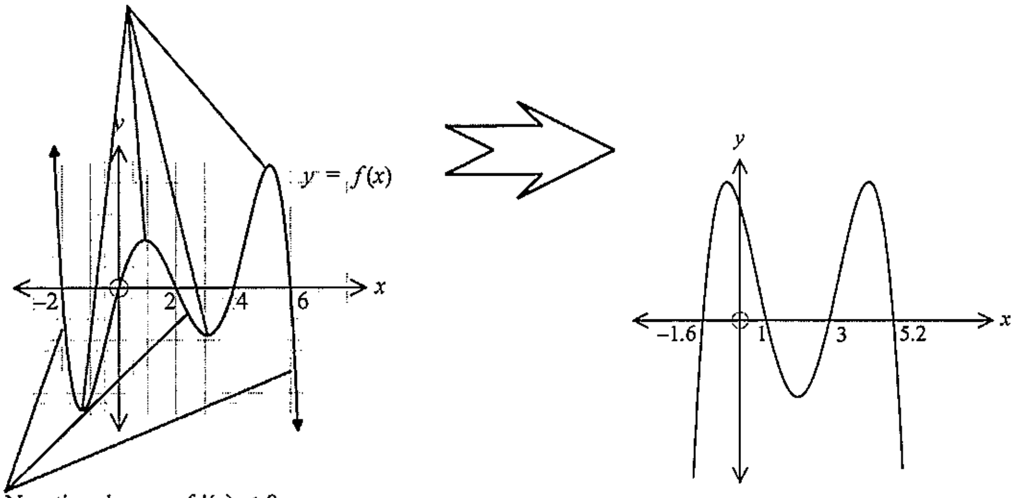
6	$\text{cosec}(\pi + \theta) = \dfrac{1}{\sin(\pi + \theta)}$ $= -\dfrac{1}{\sin\theta}$	A
7	6 of the 12 marbles are red. $P(R\,R\,R) = \dfrac{6}{12} \times \dfrac{5}{11} \times \dfrac{4}{10}$ $= \dfrac{1}{11}$	C
8	$ln(2e) = ln(2) + \ln e = ln(2) + 1$ $ln(3e) = ln(3) + \ln e = ln(3) + 1$	D

x	e	$2e$	$3e$
$\ln(x)$	1	$\ln(2) + 1$	$\ln(3) + 1$

$$\int_e^{3e} lnx\,dx \approx \frac{e}{3}(1 + 4(ln(2) + 1) + ln(3) + 1)$$
$$\approx \frac{e}{3}(1 + 4ln(2) + 4 + ln(3) + 1)$$
$$\approx \frac{e}{3}(4ln(2) + ln(3) + 6)$$
$$\approx \frac{e}{3}(ln(2^4) + ln(3) + 6)$$
$$\approx \frac{e}{3}(ln(16) + ln(3) + 6)$$
$$\approx \frac{e}{3}(ln(16 \times 3) + 6)$$
$$\approx \frac{e\,(ln(48) + 6)}{3}$$

9	Initial value is 20 so this is y intercept. $\dfrac{dA}{dt} = 0.25A$ So it is exponential growth, not decay, since constant is 0.25	C



10.	Turning points where $f'(x) = 0$  Negative slope so $f'(x) < 0$	**D**

Question 11		2016	
	Solution	Marks	Allocation of marks
(a)	$(2\sqrt{2}-\sqrt{3})(\sqrt{2}-\sqrt{3}) = 2\sqrt{4}-2\sqrt{6}-\sqrt{6}+\sqrt{9}$ $= 4-3\sqrt{6}+3$ $= 7-3\sqrt{6}$	2	2 marks for correct answer 1 mark if either the expansion or simplification are correct with an error elsewhere.

b) $\frac{a^3-b^3}{a^2-b^2} = \frac{(a-b)(a^2+ab+b^2)}{(a+b)(a-b)}$ ✓ correct factorisation of either denominator or numerator

$= \frac{a^2+ab+b^2}{a+b}$ ✓

c) $y=(x^2-2)^4$

$x=1 \Rightarrow y=(1^2-2)^4=1 \Rightarrow (1,1)$ correct differentiation

$y' = 4(2x)(x^2-2)^3$ ✓

$x \Rightarrow 1 \Rightarrow y' = m_T = 8(1)(1^2-2)^3 = -8$

$(1,1)\quad m_T=-8$

$\therefore$ eqⁿ of tangent $\Rightarrow y-1=-8(x-1)$

$y-1=-8x+8$

$8x+y-9=0$ ✓ correct

(d)	$\int_2^4 \frac{6x^4-3x^3-1}{x^2}\,dx = \int_2^4\left(\frac{6x^4}{x^2}-\frac{3x^3}{x^2}-\frac{1}{x^2}\right)dx$ $= \int_2^4 6x^2-3x-x^{-2}\,dx$ $= \left[\frac{6x^3}{3}-\frac{3x^2}{2}-\frac{x^{-1}}{-1}\right]_2^4$ $= \left(128-24+\frac{1}{4}\right)-\left(16-6+\frac{1}{2}\right)$ $= 104\frac{1}{4}-10\frac{1}{2}$ $= 93\frac{3}{4}$	3	3 marks for correct value 2 marks for a solution which includes 2 of these Correct simplification prior to integration Finding the indefinite integral Substitution into the indefinite integral to obtain definite integral 1 mark for solution which includes at least one part of the above.

		Marks	Marking criteria
(e)	$\frac{d}{dx}(\sqrt{x}.e^x) = \frac{d}{dx}\left(x^{\frac{1}{2}}.e^x\right)$ $= \left(x^{\frac{1}{2}}\right)(e^x) + (e^x)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$ $= \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)(e^x)$ $= \frac{e^x(1+2x)}{2\sqrt{x}}$ $= \frac{e^x + 2xe^x}{2\sqrt{x}}$	2	2 marks for any of the last three lines of working or other simplified equivalent expression. 1 mark for a solution which shows correct use of product rule and differentiation applied to the individual components, with a minor error in one component, or correct differentiation of components, with an error in use of product rule, or similar merit.

f) $\frac{x}{x+4.2} = \frac{5.6}{8.2}$ ✓ correct set up of ratios.

$8.2x = 5.6x + 23.52$

$2.6x = 23.52$

$x = 9.0461\ldots$ ✓ correct answer.

$x = 9.0\text{mm}$ (nearest mm)

g) $4x^2 - 5x - 1 = 0$

(i) $\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{4} = \frac{5}{4}$ ✓ (both correct)

$\alpha\beta = \frac{c}{a} = \frac{-1}{4}$

(ii) $\alpha^2\beta + \alpha\beta^2$

$= \alpha\beta(\alpha + \beta)$

$= -\frac{1}{4} \times \frac{5}{4}$

$= -\frac{5}{16}$ ✓ Correct

Question 12

a)

Criteria	Marks
• Correct solution	2
• Obtains discriminant and notes $\Delta \geq 0$	1

Sample answer

$\Delta = b^2 - 4ac$

$= m^2 - 4(m+3)$

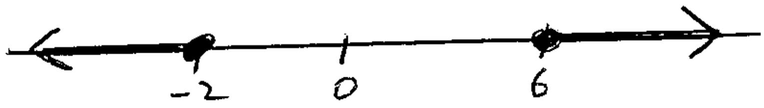
$= m^2 - 4m - 12$ ✓

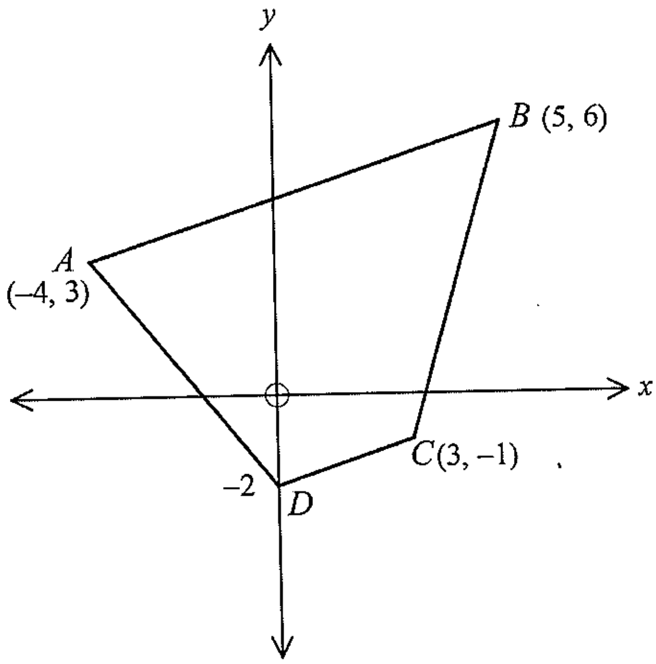
For real roots $\Delta \geq 0$

$m^2 - 4m - 12 \geq 0$

$(m-6)(m+2) \geq 0$

$m \leq -2$ or $m \geq 6$



(b) i)	 $m_{AB} = \frac{6-3}{5--4} = \frac{3}{9} = \frac{1}{3}$ $m_{DC} = \frac{-1--2}{3-0} = \frac{1}{3}$ $\therefore AB \parallel DC$ $\therefore ABCD$ is a trapezium.	2	2 marks for correct answer 1 mark if only one of the gradients is calculated correctly or if a similar error is made in the solution.

Question 12		**2016**	
	Solution	Marks	Allocation of marks
(b) ii)	Equation AB using $m_{AB} = \frac{1}{3}$ and point $(-4, 3)$ $y - 3 = \frac{1}{3}(x + 4)$ $3y - 9 = x + 4$ $x - 3y + 13 = 0$	1	1 mark for correct answer {can also use the point (6, 5)}
(b) iii)	$D = (x_1, y_1) = (0, -2)$ $d = \left\lvert \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\rvert$ $= \left\lvert \frac{1 \times 0 - 3 \times (-2) + 13}{\sqrt{1^2 + 3^2}} \right\rvert$ $= \left\lvert \frac{19}{\sqrt{10}} \right\rvert$ $= \frac{19}{\sqrt{10}}$	1	1 mark for correct answer
(b) iv)	$AB = \sqrt{(6-3)^2 + (5+4)^2}$ $= \sqrt{(3)^2 + (9)^2}$ $= \sqrt{9 + 81}$ $= \sqrt{90}$ $= 3\sqrt{10}$ $DC = \sqrt{(3-0)^2 + (-1+2)^2}$ $= \sqrt{(3)^2 + (1)^2}$ $= \sqrt{9 + 1}$ $= \sqrt{10}$ Area $= \frac{h}{2}(a + b)$ $= \frac{1}{2} \times \frac{19}{\sqrt{10}}(3\sqrt{10} + \sqrt{10})$ $= \frac{19}{2\sqrt{10}}(4\sqrt{10})$ $= 38$ sq units	2	2 marks for correct answer 1 mark if only one of the distances is calculated correctly or if an error is made in the calculation of the area.

Question 12 (c) (i)

(i)

Criteria	Marks
• Correct solution	2
• Substitutes both a and d in correct formulae	1

Sample answer

$a = 12000 \qquad d = 800$

$T_n = a + (n-1)d$

$T_{24} = 12000 + 23(800)$ ✓ correct set up of T_{24}

$= \underline{30400 \text{ phones}}$ ✓ correct answer

(ii) $S_n = \frac{n}{2}\{a + l\}$

$= \frac{24}{2}\{12000 + 30400\}$ ✓ correct set up of an expression for S_n

$= \underline{508\,800 \text{ phones}}$ ✓ correct answer

(iii) $S_{Sampson} = S_{Frontier}$

$\frac{n}{2}\{24000 + (n-1)800\} = \frac{n}{2}\{10000 + (n-1)1500\}$ ✓ correct set up equation

$n\{800n + 23200\} = n\{1500n + 8500\}$

$800n^2 + 23200n = 1500n^2 + 8500n$

$700n^2 - 14700n = 0$

$n^2 - 21n = 0$

$n(n-21) = 0$

$n = 0 \text{ or } 21$

$\therefore$ Total sales become equal after $\underline{21 \text{ months}}$. ✓

Question 13

a) $f(x) = \log_{10} 2x$

$= \dfrac{\log_e 2x}{\log_e 10}$ ✓ correct use of change of base formula.

$= \dfrac{1}{\ln 10} \ln 2x$

$f'(x) = \dfrac{1}{\ln 10} \cdot \dfrac{2}{2x}$ ✓ correct $f'(x)$

$= \dfrac{1}{x \ln 10}$

$f'(2) = \dfrac{1}{2 \ln 10}$ ✓ correct substitution

b) $y = \cos^3 3x = (\cos 3x)^3$

(i) $y' = 3(\cos 3x)^2 (-3 \sin 3x)$ ✓ correct use of chain rule

$= -9 \sin 3x \cos^2 3x$ as required.

✓ correct atleast one derivative

(ii) $\dfrac{d}{dx}(\cos^3 3x) = -9 \sin 3x \cos^2 3x$

$\int d(\cos^3 3x) = \int (-9 \sin 3x \cos^2 3x)\, dx$

$\cos^3 3x = -9 \int \sin 3x (1 - \sin^2 3x)\, dx$ — correct use of Trig Identity ✓

$-\frac{1}{9} \cos^3 3x = \int (\sin 3x - \sin^3 3x)\, dx$

correct ✓ $\therefore \int (\sin 3x - \sin^3 3x)\, dx = -\frac{1}{9} \cos^3 3x + C$

Question 13

c) (i) $P = 5000e^{kt}$

$6000 = 5000e^{k(1)}$

$e^{k} = \frac{6}{5} \Rightarrow k \ln e = \ln 6/5$

$k = \ln 6/5$

$= 0.182321\ldots$

$= 0.18$ (to 2dp) ✓ correct answer

(ii) $15000 = 5000e^{0.18t}$

$3 = e^{0.18t}$

$\ln 3 = 0.18t \ln e$

$t = \frac{\ln 3}{0.18}$

$= 6.1034\ldots$ weeks — 1 mark for correct answer in weeks ✓

= 6 days

= 43 days (nearest day)

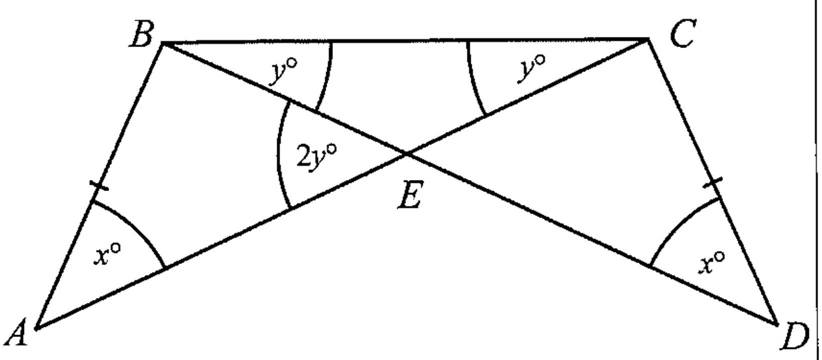
(iii) $P = 5000e^{kt}$

$\frac{dP}{dt} = 5000ke^{kt}$ — correct $\frac{dP}{dt}$ ✓

$= 5000(0.18)e^{(0.18)2}$

$= 1289.99647\ldots$ [132.72 if used $\ln 6/5$]

$\simeq 1290$ bees/week ✓ correct answer.

d)	i)  In $\Delta\, ABE$ and $\Delta\, DCE$ $\angle A = \angle D = x°$ (given) $AB = CD$ (given) $\angle BEA = \angle CED$ (vertically opposite $\angle$) $\therefore \Delta\, ABE \equiv \Delta\, DCE$ (AAS)	2	2 marks for correct proof 1 mark for an incorrect or incomplete proof with some correct and relevant statements
	ii) $BE = CE$ (corresponding sides of congruent Δs) $\angle EBC = \angle ECB = y°$ ($\Delta\, EBC$ is isosceles from above) $\angle BEA = 2y°$ (exterior angle of $\Delta\, EBC$) $\angle ABE = 180° - x - 2y$ ($\angle$ sum $\Delta\, ABE$) $\angle ABE = 180° - (x + 2y)°$	2	2 marks for correct proof 1 mark for an incorrect or incomplete proof with some correct and relevant statements

Question 14		2016	
	Solution	Marks	Allocation of marks
(a)	$\frac{dy}{dx} = 6e^{3x-6}$ $y = 6\int e^{3x-6}\,dx$ $y = 6 \times \frac{1}{3}e^{3x-6} + C$ $y = 2e^{3x-6} + C$ When $x = 2, y = 7$ $7 = 2e^{3\times 2-6} + C$ $7 = 2e^{0} + C$ $7 = 2 + C$ $C = 5$ $y = 2e^{3x-6} + 5$	2	2 marks for correct equation for y. 1 mark if valid attempt at solution which has a minor error in calculations, differentiation or algebra, or which is correct to a point but incomplete.
(b) (i)	$2u^2 + \sqrt{3}u - 3 = 0$ $u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(\sqrt{3}) \pm \sqrt{3 - 4(2)(-3)}}{2(2)}$ $= \frac{-(\sqrt{3}) \pm \sqrt{27}}{4}$ $= -\frac{\sqrt{3}}{4} \pm \frac{3\sqrt{3}}{4}$ $u = \frac{\sqrt{3}}{2}, -\sqrt{3}$	2	2 marks for 2 correct exact solutions for u. 1 mark if valid attempt at solution with an error in calculation or algebra including giving extra incorrect answers.
(ii)	$2cos^2x + \sqrt{3}\,cosx - 3 = 0$ Let $u = cos\,x$ So, from part i) $cosx = \frac{\sqrt{3}}{2},$ $x = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ or $cos\,x = -\sqrt{3}$ No solution Solutions are $x = \frac{\pi}{6}$ or $\frac{11\pi}{6}$	2	2 marks for exactly 2 correct solutions for x. 1 mark if valid attempt at solution with an error in calculation or algebra, or for answers in the wrong quadrants, including giving extra incorrect answers.

Q14

c) $3\cos 2x = -\frac{3}{2}$

(i) $\cos 2x = -\frac{1}{2}$ must show this

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}$ " " "

$x = \frac{\pi}{3}, \frac{2\pi}{3}$ " " "

$\therefore x$ coordinate of A is $\frac{\pi}{3}$

(ii) $A = \int_0^{\frac{\pi}{3}} 3\cos 2x + \frac{3}{2}\,dx$

$= \left[+\frac{3}{2}\sin 2x + \frac{3}{2}x\right]_0^{\frac{\pi}{3}}$

$= \left(+\frac{3}{2}\sin\left(\frac{2\pi}{3}\right) + \frac{\pi}{2}\right) - \left(-\frac{3}{2}\sin(0) + 0\right)$

$= +\frac{3\sqrt{3}}{4} + \frac{\pi}{2}$ units squared

d) $P = \$300\,000$, $r = 6\%\ pa = 0.5\%\ p\,month = 0.005$

$n = 25$ years $= 300$ months.

(i) $A_1 = 300000(1.005)^1 - M$

$A_2 = [300000(1.005)^1 - M]1.005^1 - M$

$= 300000(1.005)^2 - 1.005M - M$

$= 300000(1.005)^2 - M(1 + 1.005)$

$= 300000(1.005)^2 - M(1 + 1.005 + 1.005^2 + \dots 1.005^{299})$

(ii) $A_n = 300000(1.005)^{300} - M(1 + 1.005 + 1.005^2 + \dots 1.005^{299})$

$0 = 300000(1.005)^{300} - M\left(\frac{1.005^{300} - 1}{1.005 - 1}\right)$

$M = \dfrac{300\,000(1.005)^{300}}{\dfrac{1.005^{300} - 1}{0.005}}$

$= \$1932.90420\ldots$

$= \$1933$

Question 14

d) (iii)

$$0 = 300000(1.005)^n - 3866\left(\frac{1.005^n - 1}{0.005}\right)$$ ✓ correct set up

$$= 300000(1.005)^n - 773200(1.005^n - 1)$$

$$= 300000(1.005)^n - 773200(1.005)^n - 773200$$

$$473200(1.005)^n = 773200$$

$$1.005^n = \frac{773200}{473200}$$

$$\therefore n = \ln\left(\frac{773200}{473200}\right) \div \ln 1.005$$

$$= 98.449..$$

$$= 98.44 \text{ months}$$ ✓ correct answer.

$\therefore$ Lok will pay off 201.55 months earlier.

Question 15		**2016**	
	Solution	Marks	Allocation of marks
(a)	$P(x, y)$, and A (-2, 5) $AP^2 = (x - -2)^2 + (y-5)^2$ $AP^2 = (x+2)^2 + (y-5)^2$ $AP^2 = x^2 + 4x + 4 + y^2 - 10y + 25$ $AP^2 = x^2 + 4x + y^2 - 10y + 29$	1	1 mark for correct expression
	$P(x, y)$, and B (4, -7). $PB^2 = (x-4)^2 + (y - -7)^2$ $PB = \sqrt{x^2 - 8x + 16 + y^2 + 14y + 49}$ $= \sqrt{x^2 - 8x + y^2 + 14y + 65}$ Now $PA = PB$ so $PA^2 = PB^2$ $x^2 + 4x + y^2 - 10y + 29 = x^2 - 8x + y^2 + 14y + 65$ $12x - 24y - 36 = 0$ $x - 2y - 3 = 0$	2	2 marks for correct equation. 1 mark for valid attempt at a solution which has an error or is incomplete.
(b) i)	$\dddot{x} = 6t - 14$ $\ddot{x} = \frac{6t^2}{2} - 14t + C_1$ $\dot{x} = 3t^2 - 14t + C_1$ When $t = 0, \dot{x} = 8$ So $C_1 = 8$ $\dot{x} = 3t^2 - 14t + 8$ $\ddot{x} = \frac{3t^3}{3} - \frac{14t^2}{2} + 8t + C_2$ $x = t^3 - 7t^2 + 8t + C_2$ When $t = 0,\ x = -2$ $\therefore C_2 = -2$ $x = t^3 - 7t^2 + 8t - 2$	2	2 marks for correct equations for velocity and displacement 1 mark for correct integration but error made substitution 1 mark for error in integration but otherwise calculated correctly.
(b) ii)	$\dot{x} = 3t^2 - 14t + 8$ $= (3t-2)(t-4)$ $\therefore 3t - 2 = 0$ or $t - 4 = 0$ $t = \frac{2}{3}$ or $t = 4$	2	2 marks for 2 correct valu[e]s of t found from equations part(i) 1 mark if only one value found 1 mark if neither values a[re] correct but working is correct except for a mino[r] error

Q15

c) $f(x) = 1 - 3x + x^3$

(i) $f'(x) = -3 + 3x^2$

$f''(x) = 6x$

At TP $f'(x) = -3 + 3x^2 = 0$

$x^2 = 1$

$x = \pm 1$

$x = 1 \Rightarrow y = 1 - 3 + 1 = -1$ $(1, -1)$ ✓

$x = -1 \Rightarrow y = 1 + 3 - 1 = 3$ $(-1, 3)$

$x = 1 \Rightarrow f''(x) > 0$ $\therefore$ minimum at $(1, -1)$ ✓

$x = -1 \Rightarrow f''(x) < 0$ $\therefore$ maximum at $(-1, 3)$

(ii) POI $\Rightarrow f''(x) = 6x = 0$

$x = 0 \Rightarrow y = 1$

must check for concavity change ✓

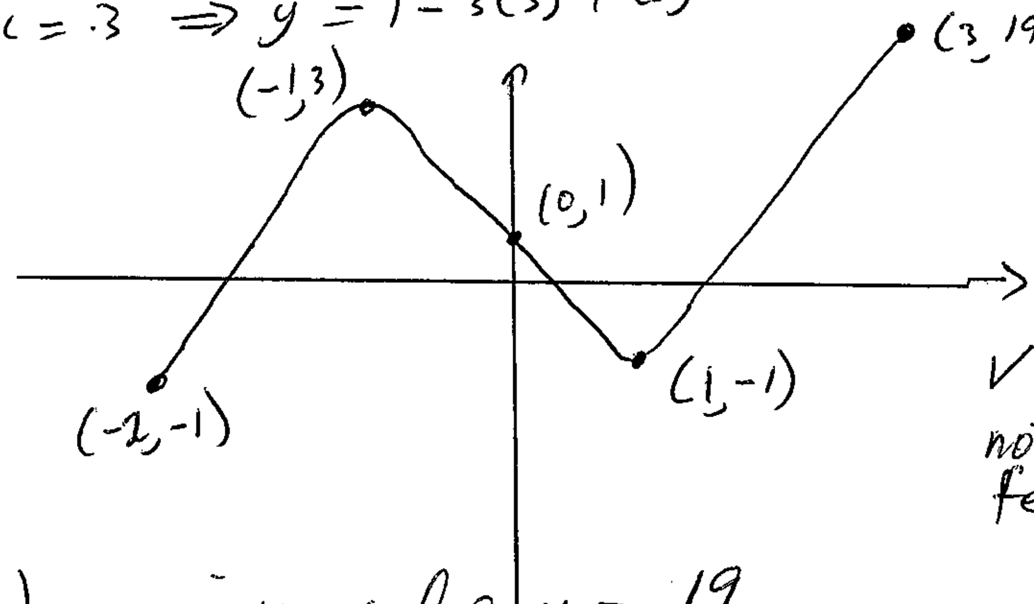
$\therefore$ POI is $(0, 1)$

x	-1	0	1
$f''(x)$	>0	0	<0

$\therefore$ Concavity changes

$x = -2 \Rightarrow y = 1 - 3(-2) + (-2)^3 = -1$ $(-2, -1)$

$x = 3 \Rightarrow y = 1 - 3(3) + (3)^3 = 19$ $(3, 19)$



✓✓ all correct

✓ if untidy sketch not indicating clearly features.

(iii) Maximum value $y = 19$ ✓

Q15.

d) $y = x \ln x$

$y' = (x)(\frac{1}{x}) + (1)\ln x$

$y' = 1 + \ln x$ ✓ correct y'

$x = e^x \Rightarrow m = y' = 1 + \ln e^x$

$= 1 + x \ln e$

$m_T = 1 + x$ ✓

Question 16

a) $y = ax^2 + bx + 4$

$y' = 2ax + b$

At intersection points: $ax^2 + bx + 4 = 2ax + b$ ✓ setting up

$x = 2 \Rightarrow 4a + 2b + 4 = 4a + b$

$\therefore b = -4$ ✓ some correct working

$x = 4 \Rightarrow 16a + 4b + 4 = 8a + b$

$\therefore a = 1$ ✓

b) (i) $y = 6$

$y = 3\sec x$

$3\sec x = 6$

$\sec x = 2 = \frac{1}{\cos x}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore$ Point of intersection A is $(\pi/3, 6)$

✓ correct trig reasoning or algebraic reasoning

✓✓ correct solution

(ii) $V = \pi \int_0^{\pi/3} (3\sec x)^2 \, dx = \pi \int 9\sec^2 x \, dx$ ✓ correct integration

$= \pi \left[9\tan x \right]_0^{\pi/3}$ correct integration ✓

$= \pi \left[9\tan \pi/3 - 9\tan 0 \right]$

$= 9\pi(\sqrt{3}) - 0$

$= 9\sqrt{3}\pi \ u^3$ ✓

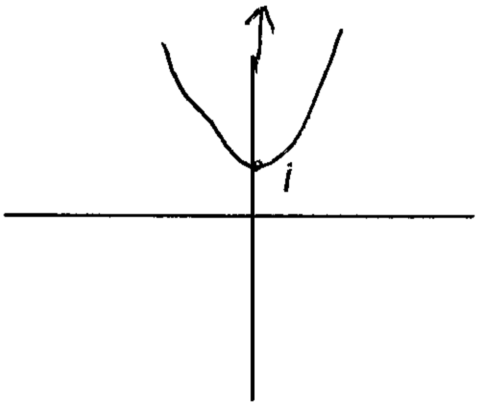
Question 16

c) $y = x^2 + 1 \Rightarrow x^2 = y - 1$

Some working showing knowledge of how to find. ✓

(i) $\therefore V(0, 1)$ $\quad 4a = 1$

$a = \frac{1}{4}$



$\therefore$ Focus $S\left(0, \frac{5}{4}\right)$ ✓

(ii) $y = x^2 + 1$

$y = x + k$

$\therefore x^2 + 1 = x + k$ ✓

$x^2 - x + (1 - k) = 0$

(iii) For one point of intersection $\Delta = 0$

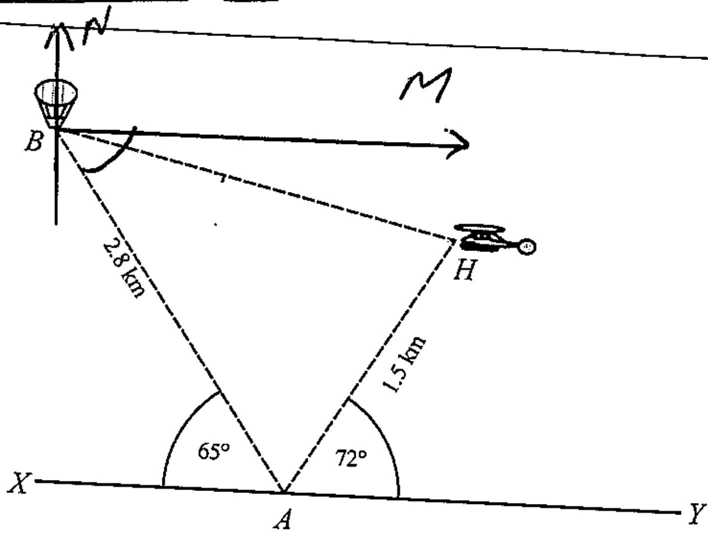
$\Delta = b^2 - 4ac = (-1)^2 - (4)(1)(1-k) = 0$

$1 - 4 + 4k = 0$

$k = \frac{3}{4}$ ✓

Question 16

d) (i)



$\angle BAH = 180 - 72 - 65 = 43°$ (supplementary ∠)

$HB^2 = BA^2 + AH^2 - 2 \times BA \times AH \times \cos \angle BAH$ (Cos Rule)

$HB^2 = 2.8^2 + 1.5^2 - 2 \times 2.8 \times 1.5 \times \cos 43°$

$= 3.9466....$

$HP = \sqrt{3.9466..}$

$= 1.9866....$

$= 2.0$ km (2 s.f)

Marks	Criteria
1	1 mark for correct working to achieve the answer required.

(ii)

In Δ BAH

$$\cos B = \frac{AB^2 + BH^2 - AH^2}{2 \times AB \times BH}$$

$$= \frac{2.8^2 + 2.0^2 - 1.5^2}{2 \times 2.8 \times 2.0}$$

$= 0.85722..$

$B = \cos^{-1}(0.85722...)$

$= 30.993826138853172244588323111713$

$= 31°$ (nearest degree) ✓

$\angle ABM = 65°$ (alternate ∠s $BM \parallel XY$)

∴ bearing $= \angle NBH$

$= 90 + 65 - 31$

$= \underline{124°}$ ✓